

NATIONAL EDUCATIONAL ASSESMENT AND EXAMINATIONS AGENCY (NEAEA)  
ETHIOPIAN UNIVERSITY ENTRANCE EXAMINATION (EUEE)  
MATHEMATICS EXAMINTION 2006

BOOKLET CODE: 00

NUMBER OF ITEMS:

SUBJECT CODE: 00

TIME ALLOWED: 00

**1. A water tank is a circular cylinder with base radius 2m and height 3m. If the tank is empty and water is pumped into it at rate of  $2\text{m}^3/\text{min}$ , how long does it take for the tank to be full?**

- A) 1.5 min  
 B)  $3/2 \pi$ min  
 C)  $6\pi$   
 D) 12 min

**2. If  $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$  and  $(2A + B)^T = A^T A$ , then which one of the following is equal to B?**

- A)  $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$   
 B)  $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$   
 C)  $\begin{pmatrix} 8 & 0 & -4 \\ 4 & 8 & 0 \\ 0 & 0 & -4 \end{pmatrix}$   
 D)  $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}$

**3. Which one of the following represents a geometric sequence?**

- A) 3, 1,  $1/3$ ,  $1/9$ ,  $1/27$ ,?  
 B)  $1/2$ ,  $-1/3$ ,  $1/4$ ,  $-1/5$ ,  $1/6$ ,?  
 C) 1, 3, 6, 10, 15,?  
 D) -3, 6, -9, 12, -15,?

4. The sequence  $\left\{ \frac{(n-1)(2n+1)}{1-n^2} \right\}_{n=1}^{\infty}$  converges to:

- A)  $-\infty$   
 B) -2  
 C) 0  
 D) 1

5. If  $F(x)$  is an ant derivative of  $f(x) = 1 - 2/x^2$  and  $F(1) = 0$ , then  $F(2)$  is equal to:

- A) 0  
 B) 1/2  
 C) -1/2  
 D) 3

6. Which one of the following expression is a polynomial expression?

- A)  $x^2 - 3x + \sin x$   
 B)  $\frac{4x^3 + 12x^2 - x}{\pi x^2}$   
 C)  $\frac{2+\pi}{1+\pi^2}$   
 D)  $2 - 3x^{\frac{2}{3}} + 7x^{\frac{5}{2}} + 3x^{-1}$

7. What is the distance from the origin to the line that passes through (1, 0) and (0, 1)?

- A)  $1/2 \sqrt{2}$   
 B) 1  
 C) 1/2  
 D)  $\sqrt{2}$

8. If the list of a measurement is 10,  $\alpha$ , 5,  $\alpha$ , 5, 10, 20, 15, 20, 5 with mean  $a$  then  $\alpha$  in terms of  $a$  is equal to:

- A)  $10a-90$   
 B)  $9a-90$   
 C)  $5a-90$   
 D)  $5a-45$

9. The total cost (in Birr) of producing  $x$  iron sheets per day is  $C(x) = 1,000 + 100x - 0.5x^2$ ,  $0 \leq x \leq 100$ . What is the marginal (rate of change of) cost at a production level of 80 iron sheets?

- A) 8.5  
 B) 20  
 C) 1,800  
 D) 5,800

10. If  $z = x + yi$  is a complex number, then  $|z|^2 + \frac{1}{2}(z - \bar{z})^2 = 1$  is equivalent to which one of the following equations?

- A)  $x^2 - y^2 = 1$   
 B)  $x^2 - 3iy^2 = 1$   
 C)  $x^2 - y^2 = 2$   
 D)  $2x^2 - y^2 = 1$

11. The following is the frequency distribution of a grouped data.

Class Intervals	Frequency (f)
3-7	2
8-12	2
13-17	10
18-22	6

What is the mean and standard deviation of the distribution, respectively?

- A)  $15, 2\sqrt{5}$   
 B)  $15, \sqrt{7.5}$   
 C)  $12.5, 5\sqrt{2}$   
 D)  $12.5, \sqrt{15}$

12. If  $(p \vee q) \Rightarrow (\neg r \wedge r)$  is true, then which one of the following is necessarily true?

- A)  $((p \vee r) \Rightarrow q)$   
 B)  $\neg p \wedge r$   
 C)  $\neg p \Rightarrow r$   
 D)  $\neg p \vee r$

13. What is the actual value of the sum  $\sum_{n=1}^{\infty} \left( \frac{2^n + 5^n}{10^n} \right)$

- A) 0.325  
 B) 1  
 C)  $5/4$   
 D)  $37/9$

14. If  $f(x) = \frac{x+1}{x-1}$  and  $f(a) = 5$  then  $f(2a)$  is equal to:

- A) 2  
 B) 4  
 C) 6  
 D) 8

15. If  $f(x) = \sqrt[3]{1 + e^{-x}}$ , which of the following is equal to  $f^{-1}(x)$ ?

- A)  $\ln\left(\frac{1}{x^3-1}\right)$
- B)  $\frac{1}{\ln(x^3-1)}$
- C)  $\ln(1-x^3)$
- D)  $(1+e^{-x})^3$

16. What is the equation of the directrix for the parabola whose equation is  $y^2 + 8x + 6y + 25 = 0$  ?

- A)  $y = 3$
- B)  $x = 2$
- C)  $x = 0$
- D)  $x = 4$

17. Given that  $\lim_{x \rightarrow 3} f(x) = 5$  and  $\lim_{x \rightarrow 3} g(x) = 11$ , what is the value of  $\lim_{x \rightarrow 3} \left( \frac{(f(x)-g(x))(g(x))}{g(x)^2-f(x)^2} \right)^n$  ?

- A) -66/96
- B) -1/16
- C) 0
- D) does not exist

18. If  $f(x) = \frac{x^2}{1+xg(x)}$ ,  $g(2) = 1$  and  $g'(2) = 10$ , then which one of the following is equal to  $f'(2)$  (the derivative of  $f$  at 2)?

- A) -8
- B) -8/9
- C) 4/3
- D) 8/9

19. If  $w = \frac{16i}{1+i} + (1-3i)^2$  and  $z = |w| + \bar{w}$ , which one of the following is the simplest form of  $z$ ?

- A)  $\sqrt{2} + 2i$
- B)  $2 + 2i$
- C)  $4 - 2i$
- D)  $2 - 2i$

20. If  $M = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -1 & 0 \\ 5 & 2 & 4 \end{pmatrix}$  and  $A^T M = 2I$ , where  $A$  is a  $3 \times 3$  matrix and  $I$  is the identity matrix of order 3, then what is  $\det(A)$ ?

- A) 0.2
- B) 4/17
- C) 0.8
- D) 1/17

21. If distinct codes ( words of eight letters are formed by rearranging the letters in the word'ABBEBAYE', how many of the codes begin with B or Y?

- A) 840  
 B) 630  
 C) 1680  
 D) 420

22. What should be the value of k so that the system of equation  $\begin{cases} x - y + z = 1 \\ -x + 5y - 4z = 1 \\ 2x + 2y - z = k \end{cases}$  has a solution?

- A) 0  
 B) 1  
 C) -4  
 D) 4

23. Let  $f(x) = \begin{cases} a \frac{\sin x}{x - |x|} & \text{if } x < 0 \\ e^{-x} + \cos x, & \text{if } x \geq 0 \end{cases}$  if f is continuous at  $x = 0$ , then what is the value of a?

- A) 4  
 B) 2  
 C) 1/2  
 D) -4

24. What is the area of the region between the graphs of  $y = -x^2 + 2$  and  $y = |x|$ , where  $-1 \leq x \leq 2$ ?

- A) 11/6  
 B) 25/6  
 C) 7/3  
 D) 11/3

25. What is the derivative of  $f(x) = \int_0^{x^2 + \pi} \frac{dt}{\sin t + 1}$  ?

- A)  $\frac{\cos x}{\sin(x^2 + \pi) + 1}$   
 B)  $\frac{2x}{\sin(x^2 + \pi) + 1}$   
 C)  $\frac{2x \cos x}{\sin(x^2 + \pi) + 1}$   
 D)  $\int_0^{2x} \frac{dt}{\sin t + 1}$

26. If two lines  $y = x$  and  $y = x - 4$  are tangent to a circle at  $(2, 2)$  and  $(4, 0)$ , respectively, then what is the equation of the circle?

- A)  $(x - 2)^2 + y^2 = 4$   
 B)  $(x - 4)^2 + (y - 2)^2 = 4$   
 C)  $(x - 3)^2 + (y - 1)^2 = 2$   
 D)  $(x - 1)^2 + (y + 1)^2 = 10$

27. Which one of the following is the set of all critical numbers of  $f(x) = \frac{1}{3}x^3 - |4x - 1|$  ?

- A)  $\{1/4, 2\}$   
 B)  $\{-2, 1/4, 2\}$   
 C)  $\{-2, 2\}$   
 D)  $\{1/4\}$

28. Which one of the following is equal to  $\int_0^{\pi/2} \frac{x - \sin x}{\sec x} dx$  ?

- A)  $\frac{\pi - 3}{2}$   
 B)  $\frac{\pi - 1}{2}$   
 C)  $\frac{3 - \pi}{2}$   
 D)  $\frac{\pi + 3}{2}$

29. Which one of the following is true?

- A) a polynomial can have infinitely many vertical asymptotes  
 B) the graph of a rational function can never cross its horizontal asymptote.  
 C) the graph of  $f(x) = \frac{3x-1}{x-1}$  has no horizontal asymptote.  
 D) the graph of  $f(x) = \frac{x^3-x}{x^2-x}$  has no vertical asymptote.

30. The simplified form of the derivative of  $f(x) = \frac{1 + \sin x}{\cos x}$  is

- A)  $\sec x + \tan x$   
 B)  $\frac{1 + \sin x}{\cos^2 x}$   
 C)  $\frac{1}{1 + \tan x}$   
 D)  $\frac{\cos x}{\sin^2 x}$

31. If  $Q_i$ ,  $D_i$  and  $P_i$  are respectively the  $i^{\text{th}}$  quartile, decile and percentile of a data arranged in an increasing order, then which one of the following is necessarily true?

- A)  $Q_2 = \frac{Q_1 + Q_3}{2}$   
 B)  $D_3 > P_{25}$

- C)  $P_{25} > Q_1$   
 D)  $Q_2 = \text{mean of the data}$

32. If  , what is the slope of the tangent line to the graph of  $f$  at  $x=2$ ?

- A) -4  
 B) 2  
 C) 18  
 D) 17

33. If  $f(x) = e^{2x} \sin x$ , then  $f''(x)$  is equal to

- A)  $3e^{2x} \sin x - 4e^{2x} \cos x$   
 B)  $4e^{2x} \sin x + 2e^{2x} \cos x$   
 C)  $e^{2x} (3\sin x + 4\cos x)$   
 D)  $e^{2x} (4\sin x - 3\cos x)$

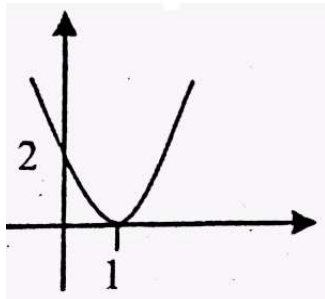
34. If the equation  $(x - 2)^2 - (y - 2)^2 = 1$  represents a hyperbola, which one of the following represents equation of an asymptote to the hyperbola?

- A)  $y = 4 - x$   
 B)  $x + y = 1$   
 C)  $x = 2 - y$   
 D)  $x + 2y = 3$

35. What is the sum of the series  $\sum_{n=1}^{\infty} (-1)^n 3^{-2n}$  ?

- A)  $-1/8$   
 B)  $-0.13$   
 C)  $-0.1$   
 D)  $1/8$

36. Which of the equations below is represented by the following parabola?



- A)  $y = x^2 + 2$   
 B)  $y = (2x - 1)^2$   
 C)  $y = 2(x - 1)^2$   
 D)  $y = (2x + 1)^2$

37. A company produced 25,000 bulbs and randomly tested 2% of the product. Among the tested bulbs, if 40 have defect of type  $D_1$ , 60 have defect of type  $D_2$  and 25 have both types of defects, what is the probability that a bulb produced by the company had none of the defects?

- A) 0.95  
 B) 0.80  
 C) 0.85  
 D) 0.20

38. A semi-elliptical arc over a tunnel for a road through a mountain has a major axis of length 80 meters and a height of 30 meters at the center. What is the equation of the semi-elliptical arc over the tunnel, if the center is considered as the origin?

- A)  $\frac{x^2}{6400} + \frac{y^2}{900} = 1$   
 B)  $\frac{x^2}{1600} + \frac{y^2}{900} = 1$   
 C)  $\frac{x^2}{900} + \frac{y^2}{6400} = 1$   
 D)  $\frac{x^2}{8100} + \frac{y^2}{6400} = 1$

39. Suppose  $AX=b$ , where  $A$  is a  $3 \times 3$  matrix,  $b=(b_1, b_2, b_3)^T$  and  $X=(x, y, z)^T$ . Which one of the following is necessarily true about this system of linear equations?

- A) The system has a solution only when  $\det(A) \neq 0$ .  
 B) the Cramer's rule is suitable to solve the system if two rows of  $A$  are identical  
 C) if  $\det(A) \neq 0$  and the second column of  $A$  is a multiple of  $b$ , then  $x = 0$   
 D) if  $b = 0$ , then  $x=(0, 0, 0)^T$  is the only solution of the system.

40. If  $y = \sin(3x^2)$ , then the simplified form of  $\frac{d^2y}{dx^2}$  is :

- A)  $-6\sin(3x^2)$   
 B)  $\cos(6x) - 6\sin(3x^2)$   
 C)  $6 \cos(3x^2) + 36x^2\sin(3x^2)$   
 D)  $x^2\cos(3x^2) + 6\sin(3x^2)$

41. If  $S$  is a set with 10 elements and  $A \subseteq S$ , what is the probability that  $A$  has 3 or more elements?

- A)  $7/10$   
 B)  $8/11$   
 C)  $121/128$   
 D)  $7/128$

42. If  $a_n = \left(\frac{n+3}{n+1}\right)^n$ , then the limit of the sequence  $\{a_n\}_{n=1}^{\infty}$  is equal to :

- A) 1

- B)  $1/2 e$   
 C)  $e^2$   
 D)  $+\infty$

43. Which one of the following is equal to  $\lim_{x \rightarrow \infty} \left(\frac{x}{x+2}\right)^{-3x}$  ?

- A)  $e^6$   
 B)  $e^{-3}$   
 C)  $e^{-3/2}$   
 D)  $e^{-6}$

44. If a box with square base open top is made from  $1,200\text{cm}^2$  material, what is the largest volume of the box in  $\text{cm}^3$  ?

- A) 4000  
 B) 8000  
 C) 15,000  
 D) 3000

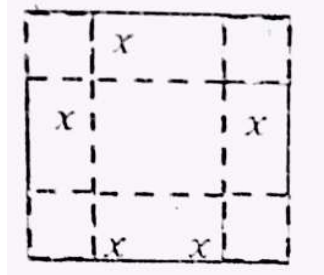
45. Which one of the following is true about the graph of  $f(x) = \frac{x^2-1}{x-x^2}$  ?

- A)  $x = 0$  and  $x = 1$  are its vertical asymptotes.  
 B)  $y = 1$  is its horizontal asymptote  
 C)  $y = x - 1$  is its oblique asymptote  
 D) it is almost the same as the horizontal line  $y = -1$  as  $x \rightarrow \pm\infty$

46. Which one of the following is a valid logical argument?

- A)  $p \Rightarrow q, q \vdash p$   
 B)  $p \Leftrightarrow q, p \Rightarrow q \vdash q$   
 C)  $\neg p \wedge q, q \Rightarrow r \vdash r$   
 D)  $\neg p, p \vee q, r \Rightarrow q \vdash r$

47. Suppose that equal squares are cut from each of the four corners of a square cardboard whose sides are 72 centimetres long. [see the figure below.] The resulting flaps are then folded up to form a box without a top. How long should be each of the four squares that has to be cut off to maximize the volume of the box?



- A) 6cm  
 B) 12cm

- C) 15cm  
 D) 24cm

48. What is the area of the region between the graph of  $f(x) = -x^2 + 4x - 3$  and the x axis from  $x = 0$  to  $x = 3$ ?

- A)  $-2/3$   
 B)  $2/3$   
 C)  $4/3$   
 D)  $8/3$

49. Which one of the following is equal to  $\int \frac{x + \ln(x+1)}{(x+1)^2} dx$  ?

- A)  $\ln(x+1) + \frac{x}{x+1} + c$   
 B)  $(x+1)^2 \frac{1}{x+1} + c$   
 C)  $(x+1)^2 - \frac{x}{x+1} + c$   
 D)  $\frac{x \ln(x+1)}{x+1} + c$

50. If  $F(x) = f(2x + 2)g(1 - x^2)$ , with  $f(2) = -3$ ,  $f'(2) = 4$ ,  $g(1) = -5$ , and  $g'(1) = -5$ , then what is the actual value of  $F'(0)$ ?

- A) -40  
 B) -20  
 C) 0  
 D) 19

51. Suppose that an airplane is descending at a speed of 50 miles per hour at an angle of  $30^\circ$  below the horizontal line. What is the x and y components, respectively of the velocity of the plane in terms of mile?

- A)  $25\sqrt{3}, 25$   
 B)  $-25, 50\sqrt{3}$   
 C)  $25, -25\sqrt{3}$   
 D)  $-25\sqrt{3}, -25$

52. If a point (2, 5) is reflected under a line to the point(-3, 1), what is the line of reflection?

- A)  $2x + 3y = 7$   
 B)  $x + 3y = 7$   
 C)  $8y + 10x = 19$   
 D)  $2x + 3y + 5 = 0$

53. Let the equation  $x^2 + 2x + y^2 = 8$  represents a circle. Then which one of the following lines cut the circle at exactly two points?

- A)  $4x + 3y + 19 = 0$   
 B)  $3x + 4y + 14 = 0$

C)  $2y = 5x + 43$

D)  $2x = y - 50$

54. Suppose  $\vec{A} = 2\vec{i} - \vec{j} + 2\vec{k}$  and  $\vec{B}$  is a vector in space such that  $|\vec{B}| = \vec{A} \cdot \vec{B}$  if  $\vec{u}$  is the unit vector in the direction of  $\vec{B}$ , then  $|\vec{A} + \vec{u}|^2$  is equal to

A) 16

B) 12

C) 10

D) 14

55. If  $A = (-2, 3)$ ,  $B = (3, 1)$  and  $C$  is any other point on the plane, then which one of the following is the coordinate form of  $\vec{AC} - \vec{BC}$  ?

A)  $(-5, 2)$

B)  $(5, -2)$

C)  $(1, 4)$

D)  $(-1, -4)$

56. What is the equation of a line that passes through the point  $(-1, -2)$  and parallel to the vector  $(1, -1)$ ?

A)  $2x - y = 1$

B)  $x + y - 1 = 0$

C)  $x - 2y = 3$

D)  $y - 2x + 1 = 0$

57. Suppose  $P$  and  $Q$  are point in space such that the midpoint of  $PQ$  is on the negative  $z$ -axis and the distance between  $P$  and  $Q$  is 6. If  $P = (2, -1, 0)$ , then what is the coordinate of  $Q$ ?

A)  $(-2, 1, 4)$

B)  $(2, -1, 6)$

C)  $(2, -1, -6)$

D)  $(-2, 1, -4)$

58. Which one of the following is equivalent to  $\neg [(\forall x)(p(x) \Rightarrow Q(x))]$  ?

A)  $(\forall x)(\neg p(x) \Rightarrow \neg q(x))$

B)  $(\exists x)(\neg p(x) \Rightarrow \neg q(x))$

C)  $(\exists x)(\neg p(x) \wedge q(x))$

D)  $(\exists x)(p(x) \wedge \neg q(x))$

59. An observer on level ground is at a distance  $10\sqrt{3}$  m from a building. The angles of elevation to the bottom of the windows on the second and third floors are  $30^\circ$  and  $60^\circ$ , respectively. What is the distance  $h$  between the bottoms of the windows?

[you may use the values:  $\sin 30^\circ = \cos 60^\circ = 1/2$  and  $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$ ]

- A) 15m
- B) 20m
- C)  $15\sqrt{3}$  m
- D) 32m

60. The following is an assertion of a person and his proof. ?For any natural numbers  $n$ ,  $n! < 10^n$ .

**Proof:**

**Step 1** let  $n=1$  Since  $1! = 1$  and  $10^1 = 10$ , it is true that  $1! < 10^1$

**step 2** let  $n=2$ , Since  $2! = 2$  and  $10^2 = 100$ , it is true that  $2! < 10^2$

**step 3** let  $n=3$ , Since  $3! = 6$  and  $10^3 = 1000$ , it is true that  $3! < 10^3$

**step 4** continuing in this manner , we can see that whenever  $k! < 10^k$  is true, then  $(k + 1)! < 10^{k+1}$  is also true.

**Therefore, by induction,  $n! < 10^n$  for all natural numbers.?**

**Which one of the following is true about the proof?**

- A) The proof is correct by the principle of mathematical induction, though Step 2 and Step 3 can be omitted.
- B) The proof is correct by the principle of mathematical induction; through Step 2 and Step 3 are necessary since they provide additional information.
- C) The proof is invalid because Step 4 did not justify the desired induction step.
- D) The proof follows the technique of a proof by exhaustion.

61. What is the image of the line given by  $(x, y) = (-1, 0) + t(3, 6)$ ,  $t \in \mathbf{R}$ , under the transition that takes  $(1, 0)$  to  $(0, 1)$  following by the reflection about the line  $y = 2x$ ?

- A)  $y = 2x + 3$
- B)  $y = 2x - 3$
- C)  $y = 2x + 6$
- D)  $y = 2x - 5$

62. If  $\theta = 2 \arctan(1/2)$ , then which one of the following is equal to  $\sec(\theta)$ ?

- A)  $35/3$
- B)  $4/5$
- C)  $5/3$
- D)  $5/4$

63. If a translation  $T$  takes the circle  $x^2 + y^2 - 2x + 6y + 3 = 0$  into the circle whose equation is  $(x + 2)^2 + (y - 4)^2 = 7$ , then what is the image of the origin under  $T$ ?

- A)  $(-3, 7)$
- B)  $(1, 2)$
- C)  $(1, -3)$
- D)  $(-2, 4)$

64. If  $L$  is the line passing through  $(0, 2)$  & parallel to  $\vec{v} = \vec{i} + 3\vec{j}$ . Which one of the following is true about  $L$  and the circle  $(x - 2)^2 + (y - 1)^2 = 5$ ?

- A)  $L$  is tangent to the circle at  $(0, 2)$

- B) L is tangent to the circle at some point P, where  $P \neq (0, 2)$ .
- C) L intersects the circle at two distinct points.
- D) the distance between L and the centre of the circle is greater than  $\neq 5$

65. Suppose  $\vec{A} = 3\vec{i} - 4\vec{j}$  and  $\vec{B}$  is a vector in the xy- plane such that the angle between  $\vec{A}$  and  $\vec{B}$  is  $\frac{\pi}{3}$ , if  $\vec{u}$  is the unit vector in the direction of  $\vec{B}$ , then  $\vec{A} \cdot (\vec{A} - 2\vec{u})$  is equal to:

- A) 20
- B) 5
- C) 15
- D) 30

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